Title: Multiplying or Dividing Rational Expressions
Class: Math 100 or Math 107 or Math 111
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Instructions to Tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: multiplying rational expressions, dividing rational expressions

Multiplying or Dividing Rational Expressions

Purpose: This is intended to refresh your skills in multiplying and dividing rational expressions.

Activity: Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

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- Multiplying or dividing rational expressions is done in the same way as multiplying or dividing fractions. However, it is generally easier to **simplify first**, before writing the answer.

- The rules for simplifying rational expressions are the same as the rules for simplifying fractions: **Only common factors may be reduced**.

- It is harder to tell when a rational expression has been **factored**:

Consider $2x^2 - x - 15$ $=$ $(2x+5)(x-3)$

The last operations are addition/subtraction. $2x^2$, $-x$, and -15 are **terms** (expressions which are added or subtracted), not factors.	The last operation is the multiplication between the)(. $(2x+5)$ and $(x-3)$ are **factors** (expressions which are multiplied).

Consider $8x^3 - 4x^2 - 60x$ $=$ $4x(2x+5)(x-3)$

$8x^3$, $-4x^2$, and $-60x$ are **terms**. These can **not** be reduced.	4, x, $(2x+5)$, and $(x-3)$ are **factors**. These can be reduced.

Example 1 $\dfrac{y^2+6y+9}{15y} \cdot \dfrac{3y^2}{2y^2+6y}$

$\dfrac{(y+3)\,\cancel{(y+3)}}{\cancel{3}\cdot 5\cdot \cancel{y}} \cdot \dfrac{\cancel{3}\cdot \cancel{y}\cdot \cancel{y}}{2\cdot \cancel{y}\cdot \cancel{(y+3)}}$

$\dfrac{(y+3)}{5\cdot 2}$

$\dfrac{y+3}{10}$

Example 2 $\dfrac{4a+12}{6a-18} \div \dfrac{5a-15}{3a+9}$

$\dfrac{2\cancel{4}\,(a+3)}{3\cancel{6}\,\cancel{(a-3)}} \cdot \dfrac{5\cancel{(a-3)}}{3\cancel{(a+3)}}$

$\dfrac{2\cdot 5}{3\cdot 3}$

$\dfrac{10}{9}$

Practice 1 $\dfrac{3p^2}{6p+24} \cdot \dfrac{9p^2+36p}{6p}$

Practice 2 $\dfrac{3x-21}{6x^2-42x} \div \dfrac{7}{12x}$

Did you get $\dfrac{3p^2}{4}$?

Did you get $\dfrac{6}{7}$?

- When terms are being added, we can rewrite their order using the commutative property of addition: $3+x = x+3$

- But, subtraction is not commutative: $3-x \neq x-3$

- However, we can factor out a -1: $3-x = -1(-3+x) = -1(x-3)$
  So, when you need to rewrite the order of two terms being subtracted, factor out a -1.

- Use the following properties to rewrite where you put your negative signs (by convention, we try not to leave any in denominators).
$$\frac{A}{-B} = \frac{-A}{B} = -\frac{A}{B} \qquad \text{and} \qquad -\frac{A}{-B} = -\frac{-A}{B} = \frac{-A}{-B} = \frac{A}{B}$$

Example 3 $\dfrac{x^2+4x+3}{x^2-4x-5} \div \dfrac{3+x}{5-x}$

$\dfrac{(x+3)(x+1)}{(x-5)(x+1)} \cdot \dfrac{-1\cdot(x-5)}{(x+3)}$

$\dfrac{\cancel{(x+3)}\,\cancel{(x+1)}}{\cancel{(x-5)}\,\cancel{(x+1)}} \cdot \dfrac{-1\cdot\cancel{(x-5)}}{\cancel{(x+3)}}$

$\dfrac{-1}{1}$

-1

Example 4 $\dfrac{c^2-5c-6}{c^2+3c} \div \dfrac{6-c}{c+3}$

$\dfrac{(c-6)(c+1)}{c\cdot(c+3)} \cdot \dfrac{(c+3)}{-1\cdot(c-6)}$

$\dfrac{\cancel{(c-6)}\,(c+1)}{c\cdot\cancel{(c+3)}} \cdot \dfrac{\cancel{(c+3)}}{-1\cdot\cancel{(c-6)}}$

$\dfrac{c+1}{-c}$

$-\dfrac{c+1}{c}$

Practice 3 $\dfrac{z^2 + 4z - 5}{25 - 25z} \cdot \dfrac{10z}{z + 5}$

Practice 4 $\dfrac{x^2 - 6x + 9}{4 - x^2} \div \dfrac{x^2 - 9}{x^2 - 8x + 12}$

Did you get $-\dfrac{2z}{5}$?

Did you get $-\dfrac{(x - 3)(x - 6)}{(x + 2)(x + 3)}$?

Problems

1) $\dfrac{x^2 + 2x + 1}{9x^3} \cdot \dfrac{2x^2 - 2x}{2x^2 - 2}$

2) $\dfrac{a^2 - 25}{8ab} \cdot \dfrac{2b}{a + 5}$

3) $\dfrac{x^2 - 1}{x^2} \cdot \dfrac{x}{1 - x} \div \dfrac{x + 1}{5}$

4) $\dfrac{2x^2 - x - 3}{x^2 - 1} \cdot \dfrac{x^2 + x - 2}{2x^2 + x - 6}$

5) $\dfrac{x^9}{x^2 - 25} \div \dfrac{x^4}{x^2 + 5x}$

6) $\dfrac{9 - a^2}{a^2 - 49} \div \dfrac{9a^2 - 27a}{3a + 21}$

7) $\dfrac{x^2 - 4}{2 - x} \div \dfrac{x^2 + 4x + 4}{2 + x}$

8) $\dfrac{c + d}{c - d} \cdot \dfrac{c - 3d}{c + 3d} \div \dfrac{c^2 - 2cd - 3d^2}{c^2 + 2cd - 3d^2}$

Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

For Tutor Use: Please check the appropriate statement:

_____Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

_____Student has mastered topic.